WIP: A Directed Graph Approach to Connecting and Assessing Critical Thinking and Problem-Solving in Engineering Education

Jaclyn E Johnson
Department of Mechanical and Aerospace Engineering
Michigan Technological University
Houghton, MI, USA
jenesbit@mtu.edu

Chang Kyoung Choi
Department of Mechanical amd Aerospace Engineering
Michigan Technological University
Houghton, MI, USA
cchoi@mtu.edu

Radheshyam Tewari
Department of Mechanical and Aerospace Engineering
Michigan Technological University
Houghton, MI, USA
rtewari@mtu.edu

Aneet D Narendranath
Department of Mechanical and Aerospace Engineering
Michigan Technological University
Houghton, MI, USA
dnaneet@mtu.edu

***Abstract*— This innovative practice work-in-progress (WIP) paper explores the interconnection between critical thinking and problem-solving in undergraduate engineering education using a unique directed graph (digraph) approach. The development and application of critical thinking and problem-solving skills are essential for fulfilling students' academic requirements and achieving success in their future careers, especially when practicing engineers tend to solve open-ended, multi-faceted, and ill-structured problems. Not only is this imperative for students' success, but also for assessing students' ability to apply critical thinking and problem-solving skills to meet ABET student outcomes, and for continuous improvement of instruction and assessment. Our work proposes the use of digraphs to quantitatively measure hierarchical components of Critical Thinking, as outlined in the discipline-neutral Paul-Elder Critical Thinking framework. Our digraph approach to connecting and assessing critical thinking and problem-solving is unique as such a quantitative approach has not been reported. We illustrate our method utilizing an example problem from a sophomore-level Introductory Thermodynamics course, where students process multiple decision-making steps in problem-solving. This decision-making approach to problem-solving can be represented as a complex and detailed digraph, reflecting the inherent nature of problems in Thermodynamics. This selection underscores our choice of course for this WIP and demonstrates the transferability of our approach to problem-solving across various engineering disciplines. This digraph approach lends to a quantitative assessment of problem-solving skills, establishing a measurable link between engineering problem-solving and the discipline-neutral Paul-Elder Critical Thinking framework, and quantitatively assessing critical thinking.**

Keywords*— *critical thinking, problem-solving, engineering curriculum, assessment, directed graphs

I. Introduction

Engineering students must develop, hone, and effectively demonstrate their critical thinking (CT) ability in the rapidly evolving landscape of engineering and technology. Not only is this imperative for students' success, but also for assessing students' ability to apply CT and problem-solving skills to meet ABET student outcomes, and for continuous improvement of instruction and assessment. According to the APA Dictionary of Psychology, critical thinking is defined as "a form of directed, problem-focused thinking in which the individual tests ideas or possible solutions for errors or drawbacks." This definition, supported by the literature [1-2], establishes a clear linkage between problem-solving and CT. There have been proposals to teach, integrate, and qualitatively assess CT via reflective writing and open-ended problems [1, 3]. However, quantitative assessment techniques for CT are conspicuous by their absence [4]. This WIP proposes a digraph based quantitative method to fill part of this gap. Establishing such an objective and quantitative method would better support operationalization of the CT theories of learning by empowering engineering educators to craft and pursue innovative pedagogical practices. Our approach is analogous to the use of reflective writing in STEM fields to express one's thought processes, potentially linking to CT on a metacognitive level [5].

We demonstrate our approach using an example engineering problem from a sophomore-level Introductory Thermodynamics course in Mechanical Engineering. In this course, students navigate through multiple decision-making steps before arriving at a solution, including identifying substance state, selecting appropriate property tables, making valid assumptions, applying mathematical interpolation, solving energy transformation equations, and reasoning the calculations. This decision-making approach to problem-solving can be represented as a complex and detailed digraph.

A digraph represents a sequence of discrete, interconnected events, with applications across various fields including social networks [5], linguistics representation [7-8], engineering problem-solving [9], and biological systems [9]. Though the underlying premise of the map is the same - with branches (edges) and nodes to map a problem-solving process - these concept maps are typically used to assess students' problem-solving process relative to an expert as opposed to the use of digraphs proposed here to connect and quantitatively assess problem-solving and CT ability. The edges in a digraph capture relationships between discrete entities. Digraphs can be quantified using measures like degree centrality, eigenvector centrality, and closeness centrality which all score the relative strength or relevance of a node.

Fig 1. illustrates a directed graph with five nodes connected by edges or arrows that connect nodes 1 to 2, 2 to 4, 4 to 3, 3 to 1, and 1 to 5, thus describing a relationship that binds the nodes in the shown sequential order. Node 1 has three edges associated with it, thereby increasing its centrality as compared to, for example, node 2 with only two edges. In this WIP we focus on the core idea of digraph representation of CT components involved in engineering problem-solving with an example drawn from a sophomore-level Engineering Thermodynamics course.

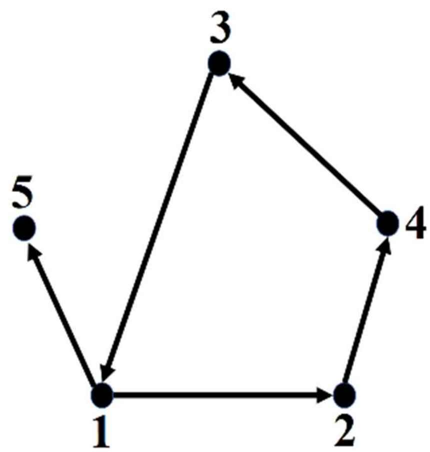

Fig. 1: Schematic illustration of a digraph (with permission from [9])

Problem-solving could lead to developing CT skills. While there are various definitions and frameworks for CT [4, 11 – 14], the author's have chosen the Paul-Elder (PE) CT framework [13]. The PE CT framework is a discipline-neutral model for CT which provides a common language and set of standards for evaluating CT. It consists of three interrelated main components viz., the Intellectual Standards (S), Elements of Thought (E), and Intellectual Traits (I). Each main component is subdivided into subcomponents, or fine-grained attributes. As depicted in [13, 15], the first main component, Intellectual Standards, includes subcomponents such as *logic*, *accuracy*, *precision*, *relevance*, *clarity*, etc. These subcomponents focus on idetifying background information and the foundation of reasoning. The second main component, Elements of Thought, has subcomponents focusing on specifying the quality of reasoning. These include, but are not limited to, *assumptions*, *implications*, *inferences*, *concepts*, *information*, and *purpose*. Lastly, the third main component, Intellectual Traits, has subcomponents focusing on key aspects and characteristics of the mind necessary for effective critical thinking. These include, among others, *intellectual integrity*, *intellectual perseverance*, *intellectual empathy*, and *confidence in reason*. Per the PE CT framework, The Standards are applied to The Elements as the Intellectual Traits are developed. Overall, the three components work together to set the foundation for, and develop, effective CT.

Literature [1-2, 16-17] shows that the PE CT framework has been utilized in engineering education to gauge students' perception of their CT ability and capacity. We propose an alternative but innovative approach involving digraphs aligned with the PE CT framework to directly and quantitatively assess CT instead of gauging students' perception. Notably, the principles discussed in [1] and [2] are in harmony with the broader frameworks and methodologies described in [15], underscoring a consolidated view of CT and problem-solving within the literature.

II. METHOD

To demonstrate a proof of concept of our overall method, we employed the following three-step approach to align CT, problem-solving, and the use of digraph representation with the PE CT framework:

1. Represent the solution steps to an engineering problem using a digraph,
2. Map the problem-solving steps to the components and subcomponents of the PE CT framework; and
3. Quantitatively assess students' CT levels using the mapped digraph of the CT components and subcomponents.

We demonstrate and discuss each step, with application to one example problem from the Introductory Thermodynamics course, in the Results and Discussion section.

III. RESULTS AND DISCUSSION

Fig. 2 shows the selected problem and solution steps used to demonstrate our method.

A. Step 1 - Represent the Solution Steps to an Engineering Problem using a Digraph

The first step involves creating a digraph illustrating the problem-solving steps - including nodes and edges for the example problem provided in Fig. 2. This requires solving, in a systematic manner, the engineering problem. Once solved, the solution can be mapped in digraph form. For this specific example, we defined two types of nodes. The type of nodes are specific to the problem solved and the engineering discipline. The first type of node (for example, the 5th and 7th nodes from top in Fig. 3), specifies problem-solving steps related to employing fundamental concepts, such as conservation of mass or energy. The second type of node (for example the 2nd node from top in Fig. 3) specifies data transformation steps, such as taking inputs of a given state (e.g., temperature, pressure, etc.) and transforming them into thermodynamic properties (e.g., enthalpy, internal energy, etc.). These nodes are intricately connected to map the problem-solving process through branches, effectively delineating the problem-solving pathway. A greater density of nodes and interconnected branches signifies a more complex problem-solving process, necessitating higher levels of CT and analytical skills from the students. The completed digraph representation of the solution steps for the example problem is shown in Fig. 3. This digraph is intentionally created in a top-down orientation to match the top-down flow of problem-solving steps for the example problem in Fig 2.

Steam enters a turbine at a pressure of 5 MPa and a temperature of 500 ^{0}C, traveling at a velocity of 60 m/s, through an inlet cross-sectional area of 0.015 m^2. It exits the turbine at a pressure of 25 kPa with a quality of 0.95, through an exit cross-sectional area of 0.14 m^2. The process operates steadily with a heat loss of 20 kJ/kg. Calculate the mass flowrate, exit velocity, and power output of the turbine.

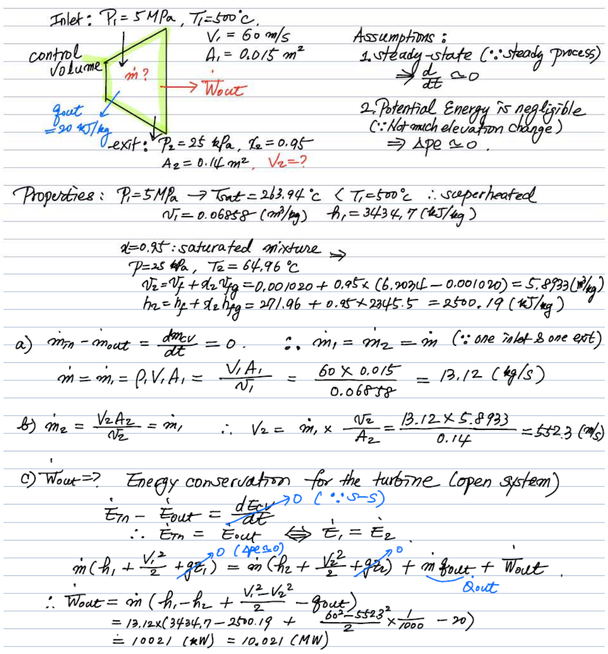

Fig. 2: Example thermodynamics problem and solution.

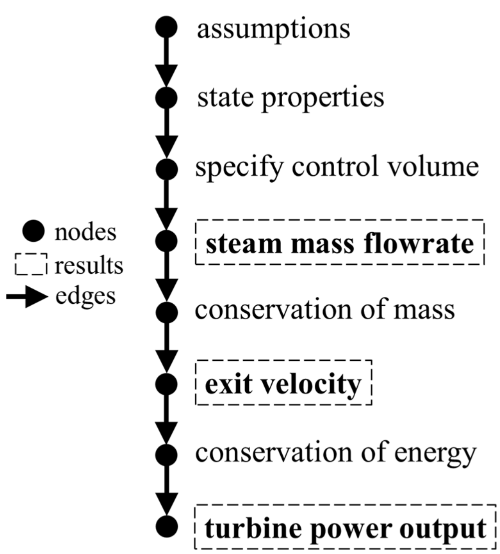

Fig. 3: Digraph representation of the problem–solving steps for the example thermodynamics problem.

B. Step 2 - Map the Problem-Solving Steps to the Components and Subcomponents of the PE CT Framework

The second step involves mapping the digraph in Fig. 3 to the PE CT framework, and subsequently transforming it into a digraph of the problem-solving process in terms of only the PE CT framework. To map the digraph in Fig. 3, the individual steps in the problem-solving process are hand-coded (labeled) with the main components, S and E, and corresponding subcomponents of the PE CT framework. S represents The Standards, and signifies problem-solving steps (nodes of the digraph) that help to outline and structure the problem solving process, including for example logic. E represents The Elements, and is used for problem-solving steps (nodes of the digraph) that require use of engineering knowledge to make assumptions and apply concepts to complete engineering calculations. For both of these components, there are subcomponents that are chosen, based on the categories of the PE CT Framework. The code nomenclature is "C - c," where the big "C" is the main component, and the little "c" is one or more of the subcomponents of the main component. For example, "E - assumptions" refers to the main component of Element of Thought and the subcomponent, or attribute "assumptions."

The big "C" codes are a broad interpretation of the CT component for that problem-solving step, while the small "c" code is a subcomponent, or fine-grained attribute, which indicates the presence of the big "C" code, or the main component. The small "c" representations assume theoretical saturation, meaning no more labels or CT codes exist beyond this. This approach - with mapping of small "c' and big "C" descriptions - aligns with traditional coding methods [18].

Fig. 4 shows the digraph in Fig. 3 mapped to the components and subcomponents of the PE CT framework. This mapped digraph includes the original labels for each node from the problem-solving process (from Fig. 3) - on the right side, and the labels of all coded components and subcomponents of the PE CT framework for each node – on the left side.

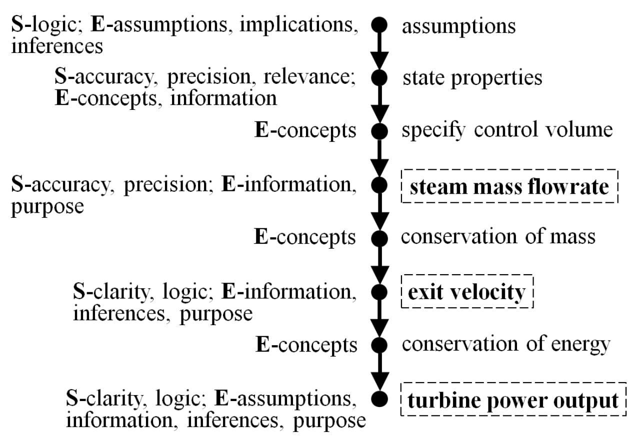

Fig. 4: PE CT framework mapped to the digraph of the solution steps for the example problem as shown in Fig. 3.

Fig. 5 shows the transformed digraph representation of the problem-solving process but only in terms of the components and subcomponents of the PE CT framework. To delineate connections between various coded S and E components and subcomponents of the PE CT framework at each node, we have added additional edges to the transformed digraph. For example, in the case of the 4th node from top, we have added two edges to show that students are expected to apply the accuracy and precision subcomponents of the component S and the information subcomponent of the component E to achieve the purpose subcomponent of the component E. Here, the E-purpose code, or label corresponds to the steam mass flowrate calculation label for the 4th node from the top in Fig. 3.

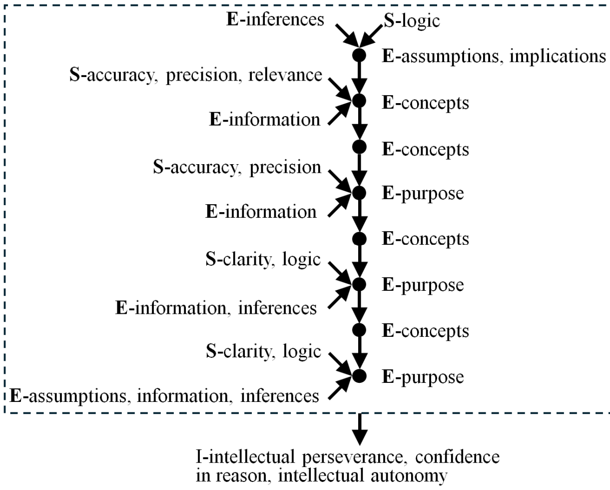

Fig. 5: Digraph representation of the problem-solving process of the example problem in terms of the PE CT framework.

The bottom most arrow coming out of the dashed line box, which encompasses the entire transformed digraph, shows our coding strategy for the I component of the PE CT framework. Specifically, with respect to our example problem, we argue that when students are successfully able to solve this problem, meaning they successfully demonstrate all coded S and E subcomponents, we can assume that they are demonstrating subcomponents such as intellectual perseverance, confidence in reason, and intellectual autonomy of the I component, or Intellectual Traits, of the PE CT framework.

C. Step 3 - Quantitatively Assess Students' CT levels using the Mapped Digraph of the PE CT Components and Subcomponents.

The third step involves application of a numeric scheme to the mapped digraph in Fig. 5 to quantitatively assess students' CT levels based on their achievement of coded components and subcomponents of the PE CT framework. We start by assigning an equally weighted point value of 1 for each subcomponent and its repetitions in the mapped digraph. This convention amounts to a total of 10 points for all unique and repeated subcomponents belonging to the S component. Similarly, it amounts to a total of 17 points for all unique and repeated subcomponents belonging to the E component. In a real assessment setting, we will compute the S and E points scored by a student and use such scores to measure their individual CT level. To elucidate, with respect to our example problem, a score in the range of 0-10 S points, normalized to 10 total points possible, expressed as a %, will be used to measure the student's demonstrated degree of possession of the S component of the PE CT framework, termed the S-measure of a student's overall CT level. Similarly, a score in the range of 0-17 E points, normalized to 17 total points possible, expressed as a %, will be used to measure the student's demonstrated degree of application of the E component of the PE CT framework, termed the E-measure of a student's overall CT level. A perfect score of 27 S and E points, which is 100% of S and E points, will be used as a measure of the student's demonstrated development of the I component of the PE CT framework.

Having devised the above-described numeric scheme for quantitatively assessing students' CT levels based on their scores of coded components and subcomponents of the PE CT framework, we now apply it to exemplify another numeric scheme to quantitatively categorize students into six thinker stages per Paul-Elder's stage theory of critical thinking [19]. Table I shows our numeric scheme to categorize students into thinker stages. The first column in Table I lists the six thinker stages, followed by the criteria column which is further divided into S and E criterion columns, with each row specifying a range of % of S or E component points for each thinker stage.

TABLE I. QUANTITATIVE CRITERIA FOR THINKER STAGES

Thinker Stages	Criteria	
	% of S Points	**% of E Points**
Unreflective	0-25%	0-25%
Challenged	>25-50%	>25-50%
Beginning	>50-75%	>50-75%
Practicing	>75-100%	>75-100%
Advanced	Frequently 100%	Frequently 100%
Master	Consistently 100%	Consistently 100%

For example, if a student's S and E component scores are in the range 0-25%, the student's performance will be categorized as being at the unreflective thinker stage. To categorize students at the advanced and master stages, we propose to consider how frequently they score 100% of the S and E points. Over the semester, if a student *frequently* scores 100% of the S and E points on various problem-solving assignments, they will be categorized at the advanced thinker stage. Frequently is defined as reaching this threshold on more than 70% of the assignments over the semester. The 70% threshold is chosen to identify students who demonstrate proficiency and a high level of competence consistently, aligning with educational grading norms. Conversely, if the student *consistently* scores 100% of the S and E scores, they will be categorized at the master thinker stage, with consistently being defined as reaching this threshold more than 95% of the assignments over the semester. This higher threshold ensures that the designation "Master Thinker" is reserved for students who demonstrate exceptional skill and consistency in their performance, signifying true mastery.

We have used professional judgement to set the % criterion for each thinker stage, however, this differentiation parallels with how thinker stages are qualitatively defined by [18] based on the variable "knowledge of thinking". We present the following hypothetical example to show parallels between our quantitative and their qualitative approaches. Imagine, with respect to our example, a student's S and E component scores, respectively, are in the range 0 - 2 points (0-25% of 10 S points) and 0 - 4 points (0-25% of 17 possible E points). Based on this, we would categorize the student to be at the unreflective thinker stage. In a real assessment setting, such a categorization would be justified because such low absolute scores of S and E components are only possible if the student failed to possess most of the S subcomponents and apply them to the E subcomponents involved in the problems-solving process. Such inferences driven by our quantitative criteria parallels with the definition of an unreflective thinker by [19], which theorizes that "…unreflective thinkers are largely unaware of thinking as such, hence they fail to recognize thinking as involving concepts, assumptions, inferences, implications, points of view, and so on. Unreflective thinkers are largely unaware of the appropriate standards for the assessment of thinking: clarity, accuracy, precision, relevance, and logicalness."

IV. Conclusions and Future work

We have applied a quantitative digraph approach connecting engineering problem-solving and CT to quantitatively assess CT skills. Our innovative method advances assessment methodologies in engineering education through the application of digraphs, offering an objective means to quantify student CT competency. This method has application to a wide-range of engineering disciplines where problem-solving is used in practice.

Future work will extend this approach to an intersection of case study and phenomenography methodologies [20], constituting a hybrid approach synergizing elements of both. This approach will allow for seamless integration of qualitative aspects of CT with quantitative analysis in engineering education research. Student-produced engineering problem-solving steps will be required for such an extension. When extending to student work, the engineering problem solving method will first need to be mapped to the PE CT framework, and then the student's solution will be scored based on the number of S and E components that are matched between the students solution and the PE CT mapped solution. This next step would allow for further evolution and development of this tool to be applied in practice.

Potential challenges and limitations are the time spent training educators, appropriate pursuit of institutional review board (IRB) exemptions or approvals, and uniform and consistent implementation of this innovative practice method amongst various educators. These challenges can be mitigated by developing detailed documentation with examples, potentially using an LMS, to help illustrate the process and train new educators on the processes, including benchmarking exercises. Additionally, though IRB approvals likely cannot be avoided, these approvals will be sought well in advance to allow for sufficient time for them to be granted. A few advantages of our work are unifying the qualitative perception of CT with quantitative metrics, generalizing engineering problem-solving and the embedded CT events using digraphs, and universalizing our approach for cross-curriculum or cross-domain implementation.

References

[1] E. Cooney, K. Alfrey and S. Owens "Critical thinking In engineering and technology education: A review," In 2008 ASEE Annual Conference & Exposition, 2008.

[2] A. Thompson, B. S. Robinson and J. C. McNeil, "Enhancing critical thinking in a first-year engineering course using a hands-on study of vectors," in 2016 ASEE Annual Conference & Exposition, 2016.

[3] L. M. Michaluk , J. Martens , R. L. Damron and K. A. High, "Developing a methodology for teaching and evaluating critical thinking skills in first-year engineering students," Int. J. Eng. Educ. 32, pp. 84–99, 2016.

[4] S. Deo and K. Hölttä-Otto, "Critical thinking assessment in engineering education: A Scopus-based literature review," ASME J. Mech. Des, vol 146(7), in press.

[5] N.B. Barr, "Revising a portfolio initiative to assess student progress in a mechanical engineering program," In 2020 ASEE North Midwest Section Annual Conference, 2020.

[6] S. Wasserma and K. Faust. Social network analysis: Methods and applications. Cambridge University Press, 1994.

[7] R. Mihalcea and P. Tarau, "TextRank: Bringing order into text," Proceedings of the 2004 Conference on Empirical Methods in Natural Language Processing, pp. 404-411, 2004.

[8] D. Paranyushkin, "Identifying the pathways for meaning circulation using text network analysis," Nodus Labs, 26, pp. 1-26, 2011.

[9] R. Tewari, A. Dharmavaram Narendranath and J. Johnson, "Work in progress: quantification of problem-complexity and problem-solving skills with directed networks in a sophomore course in mechanics of materials, in 2023 ASEE Annual Conference & Exposition, 2023.

[10] D. J. Watts and S. H. Strogatz, "Collective dynamics of 'small-world' networks," Nature, vol 393, pp. 440-442, 1998.

[11] J.C. Bean and D. Melzer, Engaging ideas: The professor's guide to integrating writing, critical thinking, and active learning in the classroom. John Wiley & Sons, 2021.

[12] L.M. Murawski, "Critical thinking in the classroom and beyond," Journal of Learning in Higher Education, 10(1), pp. 25-30, 2014.

[13] R. Paul and L. Elder, The miniature guide to critical thinking concepts and tools. Rowman & Littlefield, 2019.

[14] S. D. Schafersman, An introduction to critical thinking. https://facultycenter.ischool.syr.edu/wp-content/uploads/2012/02/Critical-Thinking.pdf, 1991.

[15] R. Paul, R. Niewoehner and L. Elder. The thinker's guide to engineering reasoning: Based on critical thinking concepts and tools. Rowman & Littlefield, 2019.

[16] K. Conn Welch, J. Hieb and J. Graham. "A systematic approach to teaching critical thinking skills to electrical and computer engineering undergraduates," American Journal of Engineering Education (AJEE) vol 6(2), pp.113-124, 2015.

[17] A. Thompson, P. A Ralston and J. L. Hieb, "Engaging freshman engineers using the Paul-Elder model of critical thinking," in 2012 ASEE Annual Conference & Exposition, 2012.

[18] D. W. Shaffer and A.R. Ruis, "How we code," in 2021 Advances in Quantitative Ethnography ICQE - Communications in Computer and Information Science, vol 1312, 2021.

[19] R. Paul and L. Elder, "Critical thinking: A stage theory of Critical thinking. Part 1," Journal of Developmental Education, 20(1), pp. 34-35, 1996.

[20] J.M. Case and G. Light, "Engineering methodologies in engineering education research," Journal of Engineering Education, vol 100 (1), pp. 186-210, 2011.